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OBSERVATION OF THE THERMAL CASIMIR FORCE IS OPEN TO QUESTION

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We discuss theoretical predictions for the thermal Casimir force and compare them with available experimental data. Special attention is paid to the recent claim of the observation of that effect, as predicted by the Drude model approach. We show that this claim is in contradiction with a number of experiments reported so far. We suggest that the experimental errors, as reported in support of the observation of the thermal Casimir force, are significantly underestimated. Furthermore, the experimental data at separations above $3\mu\text{m}$ are shown to be in agreement not with the Drude model approach, as is claimed, but with the plasma model. The seeming agreement of the data with the Drude model at separations below $3\mu\text{m}$ is explained by the use of an inadequate formulation of the proximity force approximation.

Keywords: Casimir force; thermal corrections; precise measurements.

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1. Introduction

During the last ten years much attention has been given to the Casimir force at nonzero temperature. This physical phenomenon is described by the Lifshitz theory¹ which presents the Casimir free energy and force between two parallel plates as a functional of the dielectric permittivity of plate materials calculated along the imaginary frequency axis, $\varepsilon(i\xi)$. The optical data for the complex index of refraction² extrapolated to low frequencies allow the calculation of $\varepsilon(i\xi)$ using the Kramers-Kronig relation. Surprisingly, for metal test bodies the use of most natural extrapolation by means of the Drude model was shown to be in violation of the Nernst heat theorem^{3,4} and in contradiction with experimental data.^{5–8} On the other hand, the extrapolation of the optical data below the edge of the absorption bands by means

of the plasma model, which disregards dissipation of conduction electrons, turned out to be in agreement with the Nernst theorem, and consistent with the experimental results. This created a serious problem because in accordance with classical Maxwell equations the dielectric permittivity in the quasistatic regime is inversely proportional to the frequency in accordance with the Drude model, whereas the plasma model is an approximation applicable only at sufficiently high (infrared) frequencies. Currently the use of the Drude and plasma models in the Lifshitz formula is customarily called the Drude^{9–11} and plasma^{8,12–14} model approaches, respectively.

In this paper we discuss present experimental status of the Drude and plasma model approaches to the thermal Casimir force. In Sec. 2 main characteristic features of measurements of the Casimir pressure between metal test bodies by means of a micromechanical oscillator are considered. These measurements exclude the Drude model approach but are consistent with the plasma model. Section 3 briefly reviews experiments with semiconductor^{15,16} and dielectric^{17,18} test bodies, where the Drude model approach was excluded as a description of the dc conductivity of dielectrics. In Sec. 4 experiments^{19–21} using a torsion balance and spherical lenses with centimeter-size curvature radii performed before 2010 are discussed. The first of them¹⁹ was interpreted²² as being in disagreement with the Drude model, but later this conclusion was cast in doubt.²³ Our main attention here is devoted to the recent experiment²⁴ claiming the observation of the thermal Casimir force, as predicted by the Drude model approach (see Sec. 5). We demonstrate that at separations below $3\text{ }\mu\text{m}$ the interpretation of this experiment is in fact uncertain because surface imperfections were ignored, and these are invariably present on surfaces of lenses of centimeter-size curvature radius.²⁵ At separations above $3\text{ }\mu\text{m}$ the experimental data are shown to agree with the plasma model approach, as opposed to what is claimed in Ref. 24. Section 6 contains our conclusions and discussion.

2. Experiments Between Metal Test Bodies Using a Micromachined Oscillator

These three experiments conducted with increased precision^{5–8} are independent measurements of the gradient of the Casimir force acting between a sphere of 300 or $150\text{ }\mu\text{m}$ radius and a plate of a micromechanical torsional oscillator, both covered with Au. Using the proximity force approximation (PFA), which leads to negligibly small errors for perfectly spherical surfaces of sufficiently large curvature radii, the gradient of the Casimir force was reexpressed as the Casimir pressure between two parallel plates. Different voltages were applied between the sphere and the plate, and the electric force was measured and found in agreement with exact theoretical results in a sphere-plate geometry. This was used to perform electrostatic calibrations. Specifically, the residual potential difference between the sphere and the plate in the absence of applied voltages was determined and found to be independent of separation. (The details of these experiments are described in Refs. 26 and 27.)

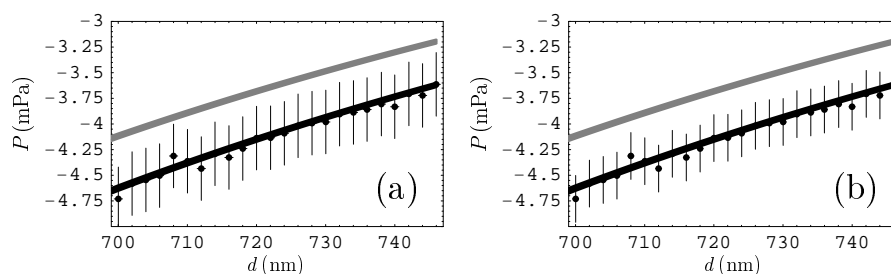


Fig. 1. The experimental data for the Casimir pressure between two parallel plates measured^{7,8} by means of micromechanical torsional oscillator as a function of separation are shown as crosses. The arms of the crosses indicate the total experimental errors determined at (a) 95% and (b) 70% confidence level. The grey and black lines show the theoretical Casimir pressures computed using the Drude and plasma model approaches, respectively. The thickness of the lines indicates the total theoretical errors.

It should be stressed that the third, and most precise, experiment using a micro-machined oscillator^{7,8} is of metrological quality, because the random error of the measured Casimir pressures was made much smaller than the systematic (instrumental) error. The comparison of the measurement results with different theoretical approaches was performed taking into account all possible undesirable systematic effects. Thus, the role of patch potentials was investigated⁶ and found to be negligibly small (here only small patches are possible with a maximum size less than the thickness of the Au layer, i.e., less than 300 nm).

The experimental results exclude the predictions of the Drude model approach over the entire measurement range from 162 to 746 nm at a 95% confidence level. The same results were found to be consistent with the plasma model approach. For the purpose of comparison with the large separation experiment²⁴ in Sec. 5, in Fig. 1 we present the measurement data indicated as crosses in comparison with theoretical predictions in the region from 700 to 746 nm. In Fig. 1(a) and 1(b) the arms of the crosses indicate the total experimental errors determined at a 95% and 70% confidence levels, respectively. As can be seen in Fig. 1, the prediction of the plasma model approach (shown by the black line) is in excellent agreement with the data, whereas the prediction of the Drude model (the grey line) is experimentally excluded. It should be stressed that the experimental and theoretical results shown in Fig. 1 are independent, and the comparison between experiment and theory has been performed with no fitting parameters.

3. Experiments with Semiconductor and Dielectric Test Bodies

The Drude-type dielectric permittivity is also used to describe dc conductivity in dielectrics and semiconductors of dielectric type. It is traditional¹ to disregard dc conductivity when dealing with the van der Waals and Casimir forces between dielectric test bodies. This was justified by the presumed smallness of this effect. It was shown,²⁸ however, that the inclusion of dc conductivity into the Lifshitz

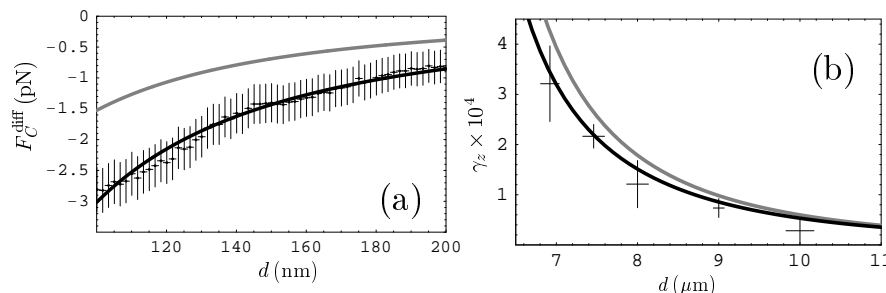
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Fig. 2. (a) The experimental data for the difference Casimir force between an Au sphere and Si plate measured^{15,16} by means of AFM as a function of separation are shown as crosses. The arms of the crosses indicate the total experimental errors determined at a 95% confidence level. The black and grey lines show the theoretical difference forces computed with neglected and included dc conductivity of Si plate in the absence of light, respectively. (b) The experimental data for the fractional change of the trap frequency due to the Casimir-Polder force between ⁸⁷Rb atoms and fused silica plate are indicated by crosses as a function of separation. The total experimental errors are determined at a 70% confidence level. The black and grey lines show the theoretical fractional shift computed with neglected and included dc conductivity of fused silica.

theory leads to a violation of the Nernst heat theorem and significantly increases the magnitude of the Casimir force. This effect was tested in the experiment^{15,16} measuring the Casimir force difference between an Au sphere of $100\ \mu\text{m}$ radius and a Si plate illuminated with laser pulses performed using an atomic force microscope. In the absence of laser light, the Si plate was in a dielectric state with a density of free charge carriers $5 \times 10^{14}\ \text{cm}^{-3}$. In the presence of light, the density of free charge carriers was increased by almost 5 orders of magnitude (semiconductor of metallic type). The experimental results for the Casimir force difference F_C^{diff} (in the presence minus in the absence of light) exclude the dc conductivity described by the Drude model at a 95% confidence level, but are consistent with the Lifshitz theory with dc conductivity in the absence of light disregarded. In Fig. 2(a) the experimental data for the difference Casimir force are indicated as crosses. The theoretical results are shown by the black and grey lines with dc conductivity in the dark phase disregarded and included, respectively. In this experiment, the experimental and theoretical results were also obtained independently, and their comparison has been made with no fitting parameters (see Refs. 26 and 27 for details).

Another important experiment is the measurement of the thermal Casimir-Polder force between ground state ⁸⁷Rb atoms, belonging to a Bose-Einstein condensate, and a fused silica plate at separations from about 7 to $10\ \mu\text{m}$. This experiment was repeated three times, in equilibrium, when the plate temperature was the same as that of environment, and out of equilibrium, when the plate temperature was higher than in the environment. In all cases the measurement data were in agreement with theory disregarding dc conductivity of fused silica,¹⁷ but were in contradiction with theory taking this conductivity into account.¹⁸ As an example, Fig. 2(b) shows the measured fractional shift of the trap frequency γ_z (crosses) due

to the Casimir-Polder force as a function of separation. The arms of the crosses are plotted at a 70% confidence level (an environment was at 310 K and the plate at 605 K). The black and grey lines show theoretical results computed disregarding and including dc conductivity of fused silica, respectively. We emphasize that, as in the previous two cases, this experiment is an independent measurement and no fitting parameters have been used when comparing the data with theory.

4. Torsion Balance Experiments

Experiments measuring the Casimir force with a torsion pendulum^{19–21,24} use the configuration of a spherical lens of greater than 10 cm radius of curvature R in close proximity to a plate. The first such experiment¹⁹ was performed in 1997 with a lens of $R = 12.5$ cm and a plate both coated with Au, and was criticized^{29,30} for overestimation of the level of agreement between the measurement data and theory. The results of this experiment at about $1\text{ }\mu\text{m}$ separation were used²² to exclude the Drude model approach to the Casimir force. At $d = 1\text{ }\mu\text{m}$ the latter predicts a -18.9% thermal correction to the force which was not observed. Recently the possibility of a systematic correction due to time-dependent fluctuations in the distance between the lens and the plate was discussed.²³ It was speculated²³ that such a correction, if it is relevant to the experiment of Ref. 19, might bring the data into agreement with the Drude model approach.

Another torsion balance experiment²⁰ used a lens of $R = 20.7$ cm curvature radius and a plate also coated with Au. The measured data over the separation region from 0.48 to $6.5\text{ }\mu\text{m}$ demonstrated a high level of agreement with the standard theory of the Casimir force and did not support the existence of large thermal corrections predicted by the Drude model. The comparison between the data and the standard theory of the Casimir force in Ref. 20 is characterized by the $\chi^2 = 513$ with the number of degrees of freedom equal to 558. From this it follows³¹ that the probability of obtaining a larger value of the reduced χ^2 in the next individual measurement is as large as 91%, that is a high level of agreement.

One more torsion balance experiment²¹ used a Ge lens and a Ge plate. The experimental precision of this experiment was not sufficient to discriminate between different theoretical approaches to the Casimir force.

The characteristic feature of the torsion pendulum experiments listed above is that all of them use fitting parameters, such as the force offset, the offset of the voltage describing a noncompensated electric force, etc. The values of these parameters are found from the best fit between the experimental data and different theories. This means that torsion balance experiments are not independent measurements, and the comparison of their results with theory is not as definitive as for the independent measurements considered in Secs. 2 and 3.

An advantage of torsion pendulum experiments is the use of lenses with centimeter-size radii of curvature, which significantly increases the magnitude of the Casimir force and allows measurements at separations of a few micrometers.

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The use of large lenses, however, leads to a problem: the experimental results in Refs. 19–21 were compared with theory using the simplest version of the PFA for the Casimir force acting between a lens and a plate²⁶

$$F(d, T) = 2\pi R \mathcal{F}(d, T), \quad (1)$$

where $\mathcal{F}(d, T)$ is the free energy (per unit area) at temperature T in the configuration of two parallel plates. Recently it was shown³² that for lenses of centimeter-size curvature radius at separations below a few micrometers, Eq. (1) is not applicable due to deviations from perfect sphericity (such as bubbles and pits) which are invariably present on lens surfaces.²⁵ For example, if there is a bubble with the radius of curvature R_1 and thickness D near the point of closest approach to the plate, the general formulation of the PFA²⁶ leads not to Eq. (1) but to the result:³²

$$F(d, T) = 2\pi(R - R_1)\mathcal{F}(d + D, T) + 2\pi R_1\mathcal{F}(d, T). \quad (2)$$

Calculations show³² that the Casimir force between a perfectly spherical lens and a plate described by the Drude model at $d < 3 \mu\text{m}$ can be made approximately equal to the force between a lens with some surface imperfections and a plate described by the plasma model, and vice versa. This makes measurements of the Casimir force by means of torsion pendulum experiments uncertain at separations below $3 \mu\text{m}$.

5. Purported Observation of the Thermal Casimir Force

Recently, one more experiment measuring the thermal Casimir force has been performed²⁴ using the torsion pendulum technique. The attractive force between an Au-coated spherical lens of $R = (15.6 \pm 0.31) \text{ cm}$ radius of curvature and an Au-coated plate was measured over a wide range of separations from 0.7 to $7.3 \mu\text{m}$. As in the case for the earlier torsion pendulum experiments mentioned in Sec. 4, the experiment of Ref. 24 is not an independent measurement. It uses two phenomenological parameters determined from the best fit between the experimental data and different theoretical approaches (see below for details). Furthermore, similar to earlier experiments exploiting large spherical lenses, the experiment of Ref. 24 ignores surface imperfections that are invariably present on surfaces of real lenses, as discussed in Sec. 4, and uses Eq. (1) in computations notwithstanding the fact that it is applicable only for perfectly spherical surfaces.

It should be emphasized that the experiment of Ref. 24 measures not the thermal Casimir force in itself, but up to an order of magnitude larger total attractive force between a lens and a plate. The total force is assumed to be the sum of the Casimir force and the electrostatic force from the large patches. As the authors themselves recognize,²⁴ “an independent measurement of this electrostatic force with the required accuracy is currently not feasible”. That is why it is hypothesized that there are large patches on Au-coated surfaces due to absorbed impurities or oxides whose size λ satisfies the condition

$$d \ll \lambda \ll r_{\text{eff}} = \sqrt{Rd}. \quad (3)$$

Note that small patches due to spatial changes in surface crystalline structure, also mentioned in Ref. 24, satisfy a condition $\lambda \ll d$ because the thickness of the Au films used is only 70 nm. The electric force due to such small patches is exponentially small.³³ Keeping in mind the parameters of the configuration used in Ref. 24, for the size of large patches satisfying Eq. (3) one obtains $\lambda \approx 50 \mu\text{m}$. The electric force due to large patches was modelled by the term²⁴

$$F_{\text{patch}}(d) = -\pi\epsilon_0 R \frac{V_{\text{rms}}^2}{d}, \quad (4)$$

where ϵ_0 is the permittivity of free space, and the parameter V_{rms} describes the magnitude of the voltage fluctuations across the test bodies. The value of this parameter was not measured and it was used as one of the fitting parameters (we assume that an attractive force is negative).

In the presence of a voltage V applied between the lens and the plate the measured force was represented in the form²⁴

$$F(d, V) = F_C(d) - \pi\epsilon_0 R \left[\frac{(V - V_m)^2}{d} + \frac{V_{\text{rms}}^2}{d} \right], \quad (5)$$

where $V_m \approx 20 \text{ mV}$ is the residual potential difference between the lens and the plate and $F_C(d)$ is the Casimir force at laboratory temperature $T = 300 \text{ K}$. It was found²⁴ that V_m is nearly independent of separation where it was determined (the variation was 0.2 mV between 0.7 and 7 μm).

The measurement data for the total force at different separations were taken with an applied potential equal to V_m in order to cancel the force from the residual potential difference. The data were corrected for the presence of fluctuations in separation and for a long-term drift of a vibration-isolation slab. The mean experimental data for the total measured force (obtained from 383 sweeps) multiplied by separations are shown on a logarithmic scale in Fig. 2 of Ref. 24 together with their errors as a function of separation. We reproduce these data in our Fig. 3 plotted on a linear scale. As can be seen in Fig. 3, the errors are unexpectedly small. For example, at the largest separation, $d = 7.29 \mu\text{m}$, the measured total force is equal to $F = (19.54 \pm 0.28) \text{ pN}$, leading to the relative error 1.4%. For the remaining 20 data points the relative errors vary from 0.86% to 2.2% (note that according to Ref. 24 the total force “is measured at 30 logarithmically spaced plate separations,” but for some unexplained reason the data at only 21 separations are shown²⁴ in Figs. 2 and 3). According to the caption to Fig. 2 in Ref. 24, “the vertical error bars include contributions from the statistical scatter of the points as well as from uncertainties in the applied corrections.” Such an important contribution of the total experimental error as a systematic (instrumental) error, which is understood as an error of a calibrated device used in force measurements (i.e., the smallest fractional division of the scale of the device), is not mentioned. The omission of a systematic error may explain the claimed smallness of all errors in Figs. 2 and 3 of Ref. 24. The point is that the absolute instrumental errors are typically constant, and this leads to a quick increase of the total relative error with increasing separation distance. If

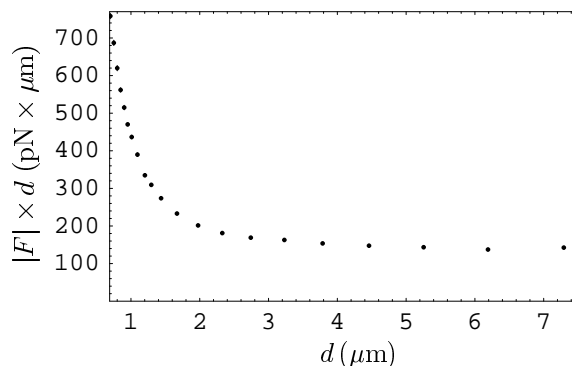


Fig. 3. The experimental data for the magnitude of the mean measured force²⁴ (electric plus Casimir) multiplied by separation are shown as crosses. The arms of the crosses indicate the experimental errors.²⁴

this is true here, it calls into question the subsequent analysis and conclusions in Ref. 24. Below, however, we attempt to follow the line of reasoning in Ref. 24 as if the errors indicated there in Fig. 2 were the total experimental errors. One should also mention that the confidence level at which the errors are found is not indicated in Ref. 24. We assume that it is on the level of one sigma.

The corrected mean data were fitted to the theoretical expression of the form²⁴

$$F(d) = F_C(d) - \pi\epsilon_0 R \frac{V_{\text{rms}}^2}{d} - a, \quad (6)$$

where in comparison with (5) one more fitting parameter a was introduced as a constant force offset due to voltage offsets in the measurement electronics.²⁴ Here, $F_C(d)$ is the theoretical thermal Casimir force which can be computed in the framework of the Lifshitz theory¹ using either the Drude or the plasma model approach. From the fitting of mean experimental data for the total force to the theoretical force in Eq. (6) with two fitting parameters V_{rms} and a , it was concluded in Ref. 24 that the data are in excellent agreement with the thermal Casimir force calculated using the Drude model approach. The plasma model was excluded in the measured separation range. Below we demonstrate that these conclusions are in fact not supported.

Computations of the thermal Casimir force within the Drude model approach were performed in Ref. 24 using the tabulated³⁴ optical data for the complex index of refraction of Au extrapolated to low frequencies by means of the Drude model. The claimed excellent agreement of these computations with the data manifested itself as the value of reduced $\chi^2 = 1.04$ with the fitting parameters $a = -3.0$ pN and $V_{\text{rms}} = 5.4$ mV. It is easily seen, however, that in the experiment under consideration (the number of degrees of freedom is equal to $21 - 2 = 19$) this value of the reduced χ^2 implies that the probability of obtaining a larger value of the reduced χ^2 in the next individual measurement is equal³¹ to 41%. For the results of an individual

measurement fitted to some model, such a χ^2 -probability could be considered as being in favor of this model. If, however, the mean measured force over a large number of repetitions is used for the fit, as in Ref. 24, the χ^2 -probability should be larger than 50% in order the measured data could be considered as supporting the theoretical model.

The next important fact is that at separations $d > 3\mu\text{m}$ the experimental data of Ref. 24 are not in agreement with the Drude model approach, as is claimed, but with the plasma model. At such large separations the theoretical predictions of the Drude and plasma model approaches to a large extent do not depend on the values of plasma frequency and relaxation parameter. Furthermore, at $d > 5\mu\text{m}$ the Casimir forces calculated using the Drude and plasma model approaches are approximately given by^{26,27}

$$F_C^D(d) = -\frac{\zeta(3)Rk_BT}{8d^2}, \quad F_C^P(d) = -\frac{\zeta(3)Rk_BT}{4d^2}, \quad (7)$$

where $\zeta(z)$ is the Riemann zeta function, i.e., the predictions of both approaches differ by a factor of two.

To compare the experimental data for the total force with the Drude and plasma model approaches at $d > 3\mu\text{m}$ (six experimental points in Fig. 2 of Ref. 24 and in our Fig. 3), we have repeated the fitting procedure to Eq. (6) with all the same corrections as were introduced by the authors. Keeping in mind that Eq. (6) contains two fitting parameters, the number of degrees of freedom is equal to four. When the Drude model approach at $T = 300\text{K}$ is used in the fit, the best agreement with Eq. (6) is achieved at $a = -0.29\text{pN}$ and $V_{\text{rms}} = 5.45\text{mV}$. The respective reduced $\chi^2 = 1.65$ leads³¹ to the probability of obtaining a larger reduced χ^2 in the next measurement equal to 16%. This signifies a poor agreement of the data with the predictions of the Drude model.

To check the predictions of the plasma model approach at $d > 3\mu\text{m}$, we used the generalized plasma-like model^{8,14,26,27} to calculate the thermal Casimir force. In this case at $T = 300\text{K}$ the best agreement between Eq. (6) and the data is achieved with $a = 3.6\text{pN}$, $V_{\text{rms}} = 4.5\text{mV}$ and the reduced $\chi^2 = 0.67$. Thus, in the next measurement a larger reduced χ^2 will be obtained³¹ with the probability 61%. This confirms that the plasma model is in a good agreement with the large separation data of Ref. 24. In Fig. 4 the magnitudes of the total theoretical forces (electric plus Casimir) multiplied by separations are shown by the grey and black lines. They are obtained from the best fit to the experimental data of Ref. 24, indicated as crosses, using the Drude and plasma model approaches, respectively. It is seen that the force data at $d > 3\mu\text{m}$ are in a good agreement with the plasma model approach.

A seeming agreement of the fit performed in Ref. 24 with the Drude model approach at separations $d < 3\mu\text{m}$ may be explained by an unjustified use of the PFA in the simplest form of Eq. (1). As explained in Sec. 4, this formulation of the PFA is not applicable to the configuration of a large lens and a plate spaced

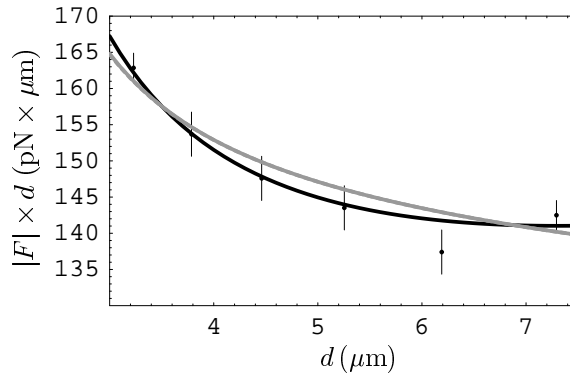


Fig. 4. The experimental data for the magnitude of the mean measured force²⁴ multiplied by separation are shown as crosses. The arms of the crosses indicate the experimental errors.²⁴ The grey and black lines demonstrate the best fit to the experimental data of the total theoretical force (electric plus Casimir) computed using the Drude and plasma model approaches, respectively, with two fitting parameters.

at separations below $3\mu\text{m}$. The surfaces of lenses with large radius of curvature ($R = 15.6\text{ cm}$ in Ref. 24) are characterized by deviations from perfect sphericity which necessitate the use of more sophisticated formulations of the PFA, such as in Eq. (2). It was shown³² that in such situations, the force with the plasma model incorporating surface imperfections can appear at separations below $3\mu\text{m}$ as if due to the Drude model.

6. Conclusions and Discussion

In the foregoing, we have discussed the comparison between experiment and theory in connection with claimed observation of the thermal Casimir force in Ref. 24, as predicted by the Drude model approach. Our first conclusion is that the experimental results of Ref. 24 are in contradiction with a number of experiments which exclude this theoretical description of the thermal Casimir force (such as the three experiments using a micromachined oscillator,^{5–8} two experiments using a torsion pendulum,^{19,20} an experiment on optical modulation of the Casimir force^{15,16} performed using an atomic force microscope, and an experiment on measuring the Casimir-Polder force^{17,18}).

Our second conclusion is that the experiment²⁴ is not an independent measurement of the thermal Casimir force, but a fit using two fitting parameters and a phenomenological expression for the electric force, presumably arising from large surface patches. According to the authors, this electric force cannot be measured with sufficient precision, although it is up to an order of magnitude greater than the thermal Casimir force. Keeping in mind that the experiments^{5–8,15–17} are independent measurements of the Casimir and Casimir-Polder force or the Casimir pressure, one may cast doubt on the results of the experiment of Ref. 24. Note also

that experiments^{19,20} use a fitting procedure similar to Ref. 24, but arrive at the opposite conclusion that the Drude model is not supported.

According to our third conclusion, the experimental errors in the mean total force in Ref. 24 are significantly underestimated. This is seen from the fact that the systematic (instrumental) errors, which are typically separation-independent, were not addressed and taken into account in the balance of errors. This resulted in surprisingly small relative experimental errors in the mean measured forces (equal to, e.g., 1.4% at the separation $7.29\text{ }\mu\text{m}$).

The fourth conclusion is that at separations above $3\text{ }\mu\text{m}$ the measurement data²⁴ are in agreement not with the Drude model approach to the thermal Casimir force, as is claimed,²⁴ but with the plasma model. This is readily demonstrated by the application of the fitting procedure²⁴ with all the same corrections, as made by the authors of Ref. 24, to the last six experimental points measured at separations above $3\text{ }\mu\text{m}$.

The last, fifth, conclusion is that a seeming agreement of the experimental data for the mean total force²⁴ with the Drude model at separations below $3\text{ }\mu\text{m}$ can be explained by the use²⁴ of an inadequate formulation of the PFA. The formulation that was employed is applicable only to perfectly shaped spherical surfaces, whereas lenses of centimeter-size radius of curvature (such as exploited in Ref. 24) invariably contain surface imperfections that must be taken into account in computations.

To conclude, the results of Ref. 24 cannot be considered as a reliable confirmation of the predictions of the Lifshitz theory combined with the Drude model. Therefore, the problem of thermal Casimir force remains to be solved.

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